

## Worksheet for Week 2 Tuesday - Limit Computing $x \rightarrow a$

**Instructions.** Discuss with your group mates and do the following problems. You are not expected to finish all the problems. :)

Steps to compute limits as  $x \rightarrow a$ :

1. "Plug in"  $\left\{ \begin{array}{l} \text{a number} \rightarrow \text{Done!} \\ \frac{0}{0} \quad \frac{\infty}{\infty} \quad \infty - \infty \quad 0 \cdot \infty \end{array} \right.$  needs more work.

2. simplify (cancel some terms)

① Fraction  $\frac{\dots}{\dots} - \frac{\dots}{\dots} \Rightarrow$  make into common denominator

② Polynomial  $\Rightarrow$  factoring

③ square root  $\sqrt{\dots} \pm \sqrt{\dots} \Rightarrow$  square root trick

1.  $\lim_{x \rightarrow 0} x^2 - 2x + 1$   
 $= 1$

2.  $\lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1}$   
 $= \lim_{x \rightarrow 1} \frac{x(x-1)}{(x-1)(x+1)}$   
 $= \frac{1}{2}$

3.  $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$   
 $= \lim_{x \rightarrow 2} \frac{\frac{2-x}{2x}}{x-2}$   
 $= \lim_{x \rightarrow 2} -\frac{1}{2x}$   
 $= -\frac{1}{4}$

Notice:  
 $2-x = -(x-2)$

$$4. \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$$

$$= \lim_{x \rightarrow 4} \frac{(x-4)}{(\sqrt{x}-2)} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{x-4} = \sqrt{4}+2 = 4$$

$$5. \lim_{y \rightarrow 0} \frac{\sqrt{4+y}-2}{y}$$

$$= \lim_{y \rightarrow 0} \frac{(\sqrt{4+y}-2)(\sqrt{4+y}+2)}{y(\sqrt{4+y}+2)} = \lim_{y \rightarrow 0} \frac{4+y-4}{y(\sqrt{4+y}+2)} = \lim_{y \rightarrow 0} \frac{y}{y(\sqrt{4+y}+2)}$$

$$= \lim_{y \rightarrow 0} \frac{1}{\sqrt{4+y}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

$$6. \lim_{x \rightarrow 2} \frac{x^2-2x}{\sqrt{x+1}-\sqrt{3}} \quad (\text{Optional - a hard problem from past exam})$$

$$= \lim_{x \rightarrow 2} \frac{x(x-2)(\sqrt{x+1}+\sqrt{3})}{(\sqrt{x+1}-\sqrt{3})(\sqrt{x+1}+\sqrt{3})}$$

$$= \lim_{x \rightarrow 2} \frac{x(x-2)(\sqrt{x+1}+\sqrt{3})}{x-2}$$

$$= 2 \cdot (\sqrt{3}+\sqrt{3})$$

$$= 4\sqrt{3}$$